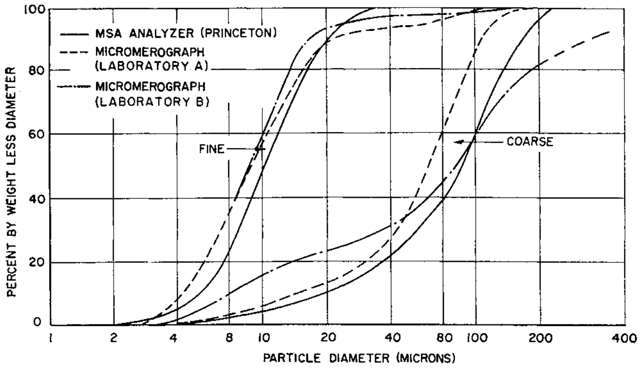

Fig. 3 Comparative particle size analyses; ground ammonium perchlorate, two grinds

is less satisfactory. However, disagreement between the two methods is not surprising since, in this size range, particles settling in air are far out of the Stokes regime, and transition from one regime to another is difficult to predict for nonspherical particles. Thus, results may be expected to vary from those from liquid sedimentation (where the behavior is entirely within the Stokes regime). The authors have no explanation to offer for the disagreement between the two Micromerograph results. (An interesting discussion of reproducibility of analyses using the Micromerograph can be found in Ref. 6.)

Liquid sedimentation analysis by the Whitby method has proved to be both a rapid and accurate method of particle size measurement for ammonium perchlorate. The technique appears superior in speed, precision, and first cost to the air sedimentation method commonly used in the solid propellant industry, but greater operational skill and judgment are required in its use.

References

[1] Bastress, E. K., "A technical report on the modification of the burning rates of ammonium perchlorate solid propellants by particle size control," Rept. 536, Dept. Aeronaut. Eng., Princeton Univ. (March 13, 1961).

[2] Bastress, E. K., Hall, K. P., and Summerfield, M., "Modification of the burning rates of solid propellants by oxidizer particle size control," ARS Preprint 1597-61 (February 1961).

[3] Orr, C., Jr. and Dalla Valle, J. M., *Fine Particle Measurement* (Macmillan Co., New York, 1959).

[4] Whitby, K. T., "A rapid general purpose centrifuge sedimentation method for measurement of size distribution of small particles," Heating, Piping Air Conditioning (June 1955).

[5] "M-S-A particle size analyzer, operating procedures and applications," Mine Safety Appliances Co., Pittsburgh, Pa.

[6] Kaye, S. M., Middlebrooks, D. E., and Weingarten, G., "Evaluation of the Sharples Micromerograph for particle size distribution analysis," Feltman Research Labs., Picatinny Arsenal TR FRL-TR-54 (February 1962).

On Mass Transfer Effectiveness

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Nomenclature

ϵ = error
R = effectiveness, $(T_w - T_c)/(T_{aw0} - T_c)$
h = heat transfer coefficient
F_c = injection ratio, $(\rho v C_p)_c/(\rho u C_p)_\infty$
ρ = density
v,u = velocities
C_p = specific heat at constant pressure
St = Stanton number, $h/(\rho u C_p)_\infty$
T = temperature

Subscripts

0 = zero injection condition
c = coolant
w = wall
aw = adiabatic wall

In a recent note the usefulness of the "effectiveness" R as a correlating parameter for data on the mass transfer cooling of turbulent boundary layers was questioned.[1] This quantity was suggested originally, though not so-called, by Friedman,[2] in a study of low-speed flow. The authors of this paper had remarkable success with it in correlating carefully taken data at Mach numbers 2 and 3.2 in flat plate flow when the coolant gas was nitrogen and the freestream gas was air.[3] When, within experimental error, later sets of data at Mach numbers 3.2 and 6.7 (Refs. 4 and 5, respectively) for a wide variety of coolant gases were found to correlate well with the previous data, the effectiveness as a "universal" parameter was proposed.[4] This must be regarded, of course, as a postulate that will be subject to the evaluation of future research.

It was recognized that the effectiveness, as previously defined,[6] has a singularity at $T_c = T_{aw0}$. Under these conditions the denominator is zero, whereas the numerator is different from zero unless for that particular case $T_c = T_w$. From the energy balance,

$$h(T_w - T_{aw}) = (C_p \rho v)_c (T_w - T_c) \qquad (1)$$

it may be seen that this exception occurs only if $T_w = T_{aw}$, which amounts to the condition $T_{aw} = T_{aw0}$, i.e., that coolant injection does not affect the recovery factor. This special case thus far has been measured only for transpiration (air into air) at zero Mach number. Thus, the existence of a singularity in R may be expected in general as T_w is varied through values in the vicinity of T_{aw0}, but in practice $T_w \ll T_{aw0}$. This singularity would not exist if one adopted a slightly different "effectiveness," namely, $R' = (T_w - T_c)/(T_{aw} - T_c)$, and in practice $R = R'$ because $|T_{aw} - T_{aw0}|/(T_{aw0} - T_c)$ is very small.

After some manipulation, the effectiveness may be expressed as

$$R = [1 + (F_c/St)(T_w/T_{aw0} - 1)/(T_w/T_{aw0} - T_{aw}/T_{aw0})]^{-1} \qquad (2)$$

For this discussion, regard St/St_0 and T_{aw}/T_{aw0} as fixed when F_c/St_0 is given, and, hence, R depends only upon T_w/T_{aw0}. T_w/T_{aw0} may vary from zero to infinity, but in practice its value generally will be less than 10^{-1}. In conducting experiments with nitrogen injection, data were obtained which yielded the two branches of the hyperbola shown for two typical conditions in Fig. 1.

The important deviation of R due to the singularity is seen to be confined to the range $0.9 < T_w/T_{aw0} < 1.1$, approximately. Furthermore, under the assumption that St/St_0 is determined uniquely by F_c/St_0, data at any one value of T_w/T_{aw0} are sufficient to determine both branches of the curve in its entirety. The assumption implies that heat transfer is proportional to $T_w - T_{aw}$, for $0 \leq T_w \leq \infty$, or, in other words, that a plot of $T_w - T_c$ vs T_w is linear, passing through zero at $T_w = T_{aw}$. This assumption may be incorrect for extremely hot or cold walls, but it is known to be correct for moderate temperature differences.

With attention confined to the neighborhood of the singularity, it is possible to insist that the forementioned assumption is valid at $T_w/T_{aw0} = 0.9$ and 1.1. Therefore, measurements made at $T_w/T_{aw0} = 1.1$ should be valid at $T_w/T_{aw0} =$

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Table 1 Comparison of Eq. (3) with data adjusted to $T_w=0$

M	$F_c \times 10^3$	$\frac{F_c}{St_0}$	$\frac{St}{St_0}$	$\frac{T_{aw}}{T_{aw0}}$	$\frac{T_w}{T_{aw0}}$	$R(T_w)$	R_E	$R(0)$	$\epsilon_1(\%)$
2.0	0.5	0.467	0.754	0.991	1.109	0.638	0.618	0.617	0.1
2.0	1.5	1.400	0.530	0.969	1.109	0.325	0.279	0.269	1.0
3.2	0.5	0.893	0.707	0.990	1.167	0.456	0.420	0.438	−1.8
3.2	1.5	2.68	0.397	0.966	1.167	0.151	0.120	0.114	0.6

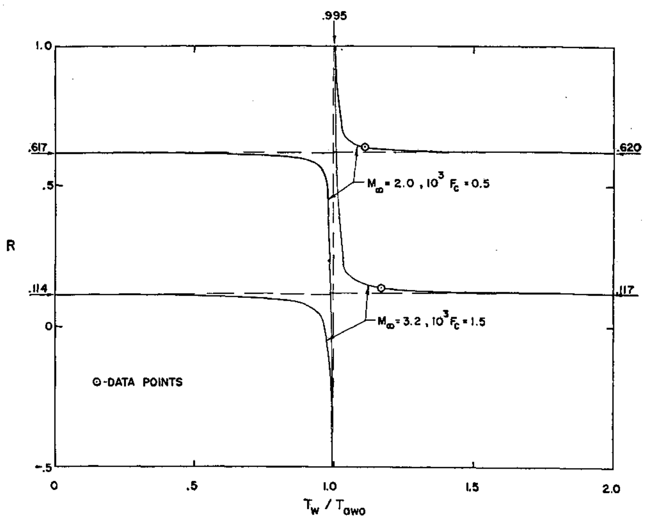

Fig. 1 The effectiveness as a function of wall temperature

0.9. Apparently also, from the example shown, the resulting R should represent well the value desired at $T_w/T_{aw0} = 0.1$. On this argument, based upon data taken with nitrogen injection, a program was planned wherein nine different coolant gases were injected. It was convenient for these latter experiments to set $T_w/T_{aw0} > 1.1$ (i.e., to preheat the "coolant" so that heat flowed from the plate to the stream). Again referring to the nitrogen injection case, one then may transcribe the data exactly to a reference value at $T_w/T_{aw0} = 0$ using Eq. (2). These points will be representative of all low T_w/T_{aw0} conditions because of the flatness of the hyperbola in this region. One may compare these transcribed data with the recommended empirical curve

$$R_E = (1 + F_c/3St_0)^{-3} \tag{3}$$

Since $R = 1$ for zero injection, one may choose unity as the proper reference quantity. Then the relative error may be taken as simply

$$\epsilon_1 = R_E - R(0) \tag{4}$$

In Table 1 these error estimates are given for four representative cases. These are comparable to, or better than, the errors accepted as tolerable in many heat transfer measurements of engineering value.

The data obtained with the various coolants scattered considerably due to the exigencies of the test situation. One would think it to be advisable for someone to repeat these measurements. Nevertheless, the results for all gases are represented well by Eq. (3), which also correlated the nitrogen data at two Mach numbers. Insofar as this formula represents all data available to date for turbulent flow over a flat plate, it properly may be called "universal," at least tentatively. If it stands the test of time, it is believed that it will be very useful in practical cases where $T_w << T_{aw0}$, despite its academically interesting singularity.

References

[1] Tewfik, O. E., "On the effectiveness concept in mass transfer cooling," J. Aerospace Sci. 29, 1382 (1962).

[2] Friedman, J., "A theoretical and experimental investigation of rocket-motor sweat cooling," J. ARS, no. 79, 147–154 (1949).

[3] Bartle, E. R. and Leadon, B. M., "The compressible turbulent boundary layer on a flat plate with transpiration cooling—I. Measurements of heat transfer and boundary layer profiles," Convair Scientific Research Laboratory Res. Rept. 11 (May 1961).

[4] Bartle, E. R. and Leadon, B. M., "The effectiveness as a universal measure of mass transfer cooling for a turbulent boundary layer," *Proceedings of the 1962 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, Calif., 1962), pp. 27–41.

[5] Danberg, J. E., private communication, U. S. Naval Ordnance Lab. (August 1961).

[6] Leadon, B. M., "Mass transfer in a turbulent boundary layer," 46th Bumblebee Aerodynamics Panel Meeting, Pomona, Calif. (May 8–9, 1962); also General Dynamics/Astronautics, Space Science Lab. Rept. ERR-AN-176 (June 25, 1962).

Application of the Mangler Transformation to a Special Class of Power Law Bodies

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Nomenclature

τ = nondimensional shear stress (defined by Pai[1])
r = body radius at any point
s = surface running length
x = distance along the axis of symmetry
n = exponent in the body expression

Subscripts

A = axially symmetric
$2D$ = two-dimensional

REFINED analyses of laminar shear stress and heat transfer can be performed by application of the Mangler transformation to the specific axially symmetric body under consideration. The present note is concerned with bodies of the $r = x^n$ class.

The following expression for the ratio of laminar shear stress on an axially symmetric body to that on a two-dimensional body has been shown[1] through the application of the Mangler transformation:

$$\frac{\tau_A}{\tau_{2D}} = \left[sr^2(s) \bigg/ \int_0^s r^2(s)ds \right]^{1/2} \tag{1}$$

Let x be introduced as the distance along the axis of symmetry. Recalling that

$$ds = [1 + (dr/dx)^2]^{1/2}dx$$

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[1] Pai, S., *Viscous Flow Theory: Laminar Flow* (D. Van Nostrand Co. Inc., Princeton, N. J., 1956), Vol. I, Chap. 11, p. 264.